GEOMETRIC PONTRYAGIN MAXIMUM PRINCIPLE FOR DISCRETE TIME OPTIMAL CONTROL PROBLEMS

MISHAL ASSIF P K

Extended abstract

We establish a geometric version of the discrete-time Pontryagin maximum principle for optimal control problems on finite dimensional smooth manifolds subject to the following three types of constraints: (a) pointwise constraints on the states, (b) pointwise constraints on the control actions, and (c) constraints on the frequency spectrum of the optimal control trajectories.

Constraints on the states (as in (a)) are desirable and/or necessary in most applications, and the class of constraints treated in our work are capable of describing a rather general family of path-planning objectives, and subsumes both ballistic and servomechanism reachability problems under constraints on the control magnitudes (as in (b)). Constraints on the control frequencies (as in (c)) are especially important in practice, especially for applications involving mechanical systems, since the traditional methods for control synthesis may lead to controls with high frequency components that cannot be faithfully reproduced by inertial actuators. However, there have been few systematic studies on control under mixed time-domain and frequency-domain constraints apart from the recent work [PC17] that is limited to systems with Euclidean configuration spaces. Here we treat optimal control problems under constraints of the type (a)-(c) on smooth manifolds.

The proof of our result proceeds as follows: First, the Whitney embedding theorem is employed to embed the smooth manifold in a finite dimensional Euclidean space. Second, the original constrained optimal control problem is extended to an equivalent one defined on the ambient Euclidean space. First order necessary conditions for discrete-time optimal control problems on Euclidean spaces under the three types of constraints (a)-(c) have recently appeared in the literature. In the third step we employ these first-order necessary conditions to the problem obtained at the end of the second step; this yields coordinate-dependent first-order necessary conditions for optimality under the preceding constraints. Finally, the necessary conditions obtained in the third step are lifted back to the manifold in a natural way to arrive at a purely geometric description of the first-order necessary for optimality of the original problem.

It is well-known that discrete-time versions of the Pontryagin maximum principle on Euclidean spaces are quite different from their continuous-time counterparts, and the same holds on smooth manifolds. Our proof follows, in spirit, the path to establish geometric versions of the Pontryagin maximum principle on smooth manifolds indicated in [Cha11] in the context of continuous-time optimal control.

Key words and phrases. discrete-time optimal control, geometric Pontryagin maximum principle, smooth manifolds.

Emails: mishal_assif@iitb.ac.in.

The recent preprint [KG17] also establishes a geometric discrete-time version of this principle under weaker hypotheses in the absence of control frequency constraints; however, the approach there relies heavily on tools from non-smooth analysis, and is not as simple as the proof that we present here.

References

[Cha11] D. E. Chang, *A simple proof of the Pontryagin maximum principle on manifolds*, Automatica **47** (2011), 630–633.

[KG17] R. Kipka and R. Gupta, *The dicrete-time geometric maximum principle*, https://arxiv.org/abs/1707.03873, 2017.

[PC17] P. Paruchuri and D. Chatterjee, *Discrete time Pontryagin maximum principle for optimal control problems under state-action-frequency constraints*, under review, preprint available at https://arxiv.org/abs/1708.04419, 2017.