Gradations, Grassmann algebras, and solutions to the classical Yang–Baxter equation

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A *coboundary Lie bialgebra* is a Lie algebra $\mathfrak{g}$ equipped with a map $\delta : v \in \mathfrak{g} \to [v, r]_S \in \Lambda^2\mathfrak{g}$, where $[\cdot, \cdot]_S$ is the Schouten bracket on the Grassmann algebra $\Lambda\mathfrak{g}$ and $r \in \Lambda^2\mathfrak{g}$ is a solution of the *modified classical Yang–Baxter equation* (MCYBE), i.e. $[v, [r, r]_S]_S = 0$ for any $v \in \mathfrak{g}$. Coboundary Lie bialgebras play an important role in mathematical and theoretical physics, e.g. in quantum group theory or integrable systems. The classification and properties of solutions of the MCYBE are well-studied mostly for semisimple Lie algebras or when $\dim \mathfrak{g} \leq 3$. In this talk, I will discuss a new geometric technique to find solutions of the MCYBE, which uses gradations on $\mathfrak{g}$ and $\Lambda\mathfrak{g}$. It allows us to tackle non-semisimple and higher-dimensional cases. To illustrate the method, several such examples will be presented.