Abelian gauge theory on noncommutative $\mathbb{R}^3$ is a scalar theory on the Moyal plane

Leonardo Santilli*
Grupo de Física Matemática
Universidade de Lisboa

March 19, 2018

Abstract

Since the very beginning, noncommutative geometry found broad resonance in physics [1]. Indeed, while the quantization process does not allow position and momentum to commute, it was easy to imagine a generalisation in the sense that, at some fundamental level, the spacetime coordinates do not commute. On the other hand, noncommutative field theories appear to be tightly related to string theories, allowing to face the calculation for stringy models in a more suitable framework.

The first example of noncommutative space is provided by the Moyal plane, which is in a certain sense the easier but the most general one. Both scalar and gauge theories, with or without matter, have been thoroughly studied on the Moyal plane. Throughout this work, we focused on the famous papers by Langamnn–Szabo–Zarembo (LSZ) [2,3], in which a scalar field theory on on the Moyal plane in presence of a background magnetic field is developed. This model is exactly solvable.

On the other hand, in recent years, a series of papers by Geré, Vitale, Wallet et al. [4–7] investigated the properties of a noncommutative version of $\mathbb{R}^3$, which admits a natural foliation in terms of fuzzy spheres. They showed how abelian gauge theories in such space can be reduced to a scalar theory on each fuzzy sphere. The formal similarity of this reduced model with the LSZ was claimed but not exploited. In particular, although the action of the two systems seemed to be the same, the content of the respective kinetic operators was rather different.

In the present work, the authors modified the geometry of noncommutative $\mathbb{R}^3$ and obtained a gauge theory which, when restricted to a specific fuzzy sphere of the foliation, reduces exactly to the LSZ scalar theory. Remarkably enough, this procedure seems to work also with different (abelian) constructions. In future work, our aim is to generalise and systematise these features, in order to allow the solution of gauge theories in 3d (noncommutative) spaces in terms of their foliation, as scalar theory on a (noncommutative) plane.

*lsantilli@fc.ul.pt

References

[1] R. J. Szabo. Quantum field theory on noncommutative spaces. *Phys. Rep.*, 378(4):207–299, 2003.

[2] E. Langmann, R. J. Szabo, and K. Zarembo. Exact solution of noncommutative field theory in background magnetic fields. *Phys. Lett. B*, 569(1-2):95–101, 2003.

[3] E. Langmann, R. J. Szabo, and K. Zarembo. Exact solution of quantum field theory on noncommutative phase spaces. *J. High Energy Phys.*, (1):017, 69, 2004.

[4] P. Vitale and J.-C. Wallet. Noncommutative field theories on $\mathbb{R}^3_\lambda$: towards UV/IR mixing freedom. *J. High Energy Phys.*, (4):115, front matter + 35, 2013.

[5] A. Géré, P. Vitale, and J.-C. Wallet. Quantum gauge theories on noncommutative three-dimensional space. *Phys. Rev. D*, 90:045019, Aug 2014.

[6] A. Géré, T. Jurić, and J.-C. Wallet. Noncommutative gauge theories on $\mathbb{R}^3_\lambda$: perturbatively finite models. *J. High Energy Phys.*, (12):045, front matter+28pp, 2015.

[7] J.-C. Wallet. Exact partition functions for gauge theories on $\mathbb{R}^3_\lambda$. *Nuclear Phys. B*, 912:354–373, 2016.