Exact discrete lagrangian for nonholonomic mechanics: an open problem

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The existence of an exact discrete lagrangian function for nonholonomic systems is still an open problem in the field of geometric integration (see [1] for the unconstrained case). In the last few decades, an effort has been made to introduce geometric numerical methods, such as variational integrators, which preserve geometric structure. In the case of variational integrators, we discretize the lagrangian function to which we apply a discrete variational principle to obtain the discrete-time equations of motion, whose solutions are sequences of points which approximate the solution for the continuous-time problem.

In this talk we will restrict to nonholonomic mechanics (See [2], [4] for an introduction to constrained systems). After discussing the associated discrete descriptions at our disposable in the existing literature (cf. [5], [7] or [6]), we introduce the problem of finding an exact discrete lagrangian function for nonholonomic mechanical systems. We will unveil the exact discrete space where nonholonomic dynamics takes place and explicitly define the corresponding exact retraction, using techniques described in [3]. Then we expose the geometric construction of the exact discrete integrator.

The discovery of the nonholonomic discrete exact lagrangian function will make an advance to the study of error analysis of numerical methods and would have many applications on subjects such as optimal control.

References

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[1] A. Simoes acknowledges FCT for the PhD research fellowship SFRH/BD/129882/2017.

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