INTEGRABILITY AND NONHOLONOMIC SYSTEMS WITH SYMMETRY

NICOLA SANSONETTO

Nonholonomic mechanical systems are mechanical systems with (nonintegrable) constraints in the velocities. A first consequence of the presence of the constraints is the non variational origin of these systems. This fact has strong implications: for example, Noether's Theorem does not hold and they are not Hamiltonian, therefore all the well developed theory of integrable Hamiltonian systems does not apply. These aspects, among others, make nonholonomic systems extremely interesting, rich and challenging.

In the course we will first review some basic facts about integrability of Hamiltonian and non–Hamiltonian systems, underlying the typical geometric features of the Hamiltonian framework that still survive in the non-Hamiltonian case.

Then, after an introduction to nonholonomic systems, we will analyze the failure of Noether's Theorem and its consequences from the point of view of integrability. Nevertheless we will discuss a method to obtain first integrals out of the symmetry of a given system, thus shedding some light to a possible relation between the existence of first integrals and the presence of symmetries in noholonomic mechanical systems.

Eventually we will apply this theory to integrable nonholonomic systems, illustrating the obtained results to concrete examples.