Geometric Integrators: Composition, Splitting and Lie Group Methods

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The numerical integration of differential equations has experienced an important progress during the las few decades. In particular, the *Geometric Integration* (GI), a new branch in Applied Mathematics, as emerged.

The evolution of many physical systems is modelled by differential equations which retain the observed qualitative properties of the systems. These equation, in general, have to be numerically integrated but, frequently and contrarily to the models, standard numerical integrators do not preserve these qualitative properties. Standard methods produced in many cases wrong qualitative results or lead to inefficient algorithms. It has been widely recognized that the class of numerical integrators which preserve the geometric properties of the exact flow provide a better description of the system. A widely used technique in GI is to compose one or more low-order basic one-step methods (usually first or second order) with appropriately chosen weights to achieve a higher order scheme. The resulting composition method then inherits the relevant geometric properties the basic scheme shares with the exact solution. In close connection with composition methods are the splitting methods (closely related with symplectic integrators), frequently used in celestial mechanics, quantum mechanics, molecular dynamics, accelerator physics and, in general, for solving numerically Hamiltonian systems, as well as Poisson systems and reversible differential equations [1, 3, 4]. Another important family of geometric integrators corresponds to the Lie group methods. These methods were mainly addressed for non-autonomous linear systems whose solution evolve in a Lie group [2], but have also been extended to non-linear systems.

In this talk we briefly introduce the GI methods and we focus in the State of the Art about composition, splitting and Lie group methods, as representative of them.

References

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