Nonlinear Stability of Hamiltonian Relative Equilibria in Simple Mechanical Systems

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The treatment of the stability of steady solutions (relative equilibria) for symmetric Hamiltonian systems has a long history starting in the early days of analytical mechanics, and more recently using techniques from symplectic geometry and Hamiltonian Lie group actions. Most of the modern available methods for testing the nonlinear stability of relative equilibria are based on energy and momentum confinement, and are designed for general Hamiltonian systems having as phase spaces arbitrary symplectic manifolds. However most Hamiltonian systems relevant in classical and quantum mechanics are "simple mechanical systems". That is, Hamiltonian systems of the form "kinetic + potential energy" defined on the cotangent space of a configuration space. In this talk we present a method for testing the stability of relative equilibria in this class of systems which incorporates in its design the fibered geometry of the phase space, as well as the separability of the Hamiltonian. The method produces a test that can be performed at the level of the configuration space only, therefore reducing the dimension of the problem by a factor of two, which can lead to important simplifications when dealing with complex systems.