Reduction of Almost Poisson Brackets for Mechanical Systems with Nonholonomic Constraints

Luis García-Naranjo

Abstract

In mechanics, constraints that restrict the possible configurations of the system are termed holonomic. A simple example is the fixed length of the rod of a pendulum. Mechanical systems with constraints on the velocities that do not arise as constraints on positions are called nonholonomic. These often arise in rolling systems, like a sphere rotating without slipping on a table.

The study of nonholonomic mechanical systems is challenging because the equations of motion are not Hamiltonian. The dynamics of the system can however be described in terms of a bracket of functions that fails to satisfy the Jacobi identity. One now speaks of an *almost Poisson bracket*. This approach encodes the forces of constraint and avoids dealing with Lagrange multipliers.

We study the reduction of nonholonomic systems with symmetry from the almost Poisson perspective. I will describe different scenarios for the reduction and discuss how for some important examples the reduced bracket satisfies the Jacobi identity (sometimes after a time reparametrization) so the reduced system can be cast in Hamiltonian form.