11th Young Researcher Workshop on Geometry, Mechanics and Control

Programme

COURSES

• Nicola Sansonetto (Università di Padova, Italy): Integrability and Nonholonomic Systems with Symmetry
• Marco Caponigro (CNAM, Conservatoire National des Arts et Métiers, Paris, France): Stabilization and optimal control in multiagent dynamics
• Alejandro Cabrera (UFRJ, Rio de Janeiro, Brasil): Introduction to supergeometry

SHORT TALKS

• Fabrizio Boriero: "Trajectory generation for under-actuated mechanical systems: the "limbo dance" problem"
• Damien Bouloc: "Singular fibers of the Gelfand-Cetlin system on $\mathfrak{su}(n)$"
• William Clark: "Time minimal control for a quantum Hamiltonian under Lindbland dissipation"
• Marine Fontaine: "Explicit symmetry breaking for equivariant Hamiltonian systems"
• Víctor Jiménez: "Study of Cosserat media with Lie groupoids and algebroids"
• Maximiliano Palacios: "A variational formalism for finite dimensional thermo-mechanical systems"
• Ana Rojo: "Discrete moving Frames and Noether's Finite Difference Conservation Laws"
• Amna Shaddad: "Point vortices on the complex projective plane"

POSTER SESSION

• Paz Albares Vicente: "Lump solitons in a generalized nonlinear Schrödinger equation in 2+1 dimensions"
• Philip Arathoon: "The dynamics of a rigid body immersed in a fluid"
• Nicolás Borda: "Discrete cotangent flows by blowing up at zero time step"
• Roksana Brodnicka: "Dynamical systems with multiplicative perturbations in financial mathematics"
• Daniel de la Fuente: "Completeness of electromagnetic trajectories in stationary spacetimes"
• Wojciech Fabjanczuk: "Superlinear extensions of linear algebra"
• Eduardo Fernández Saiz: "Quantum algebra in Lie-Hamilton system"
• Iván Gutiérrez Sagredo: "Drinfel'd doubles: the vanishing cosmological constant case"
• Juan Jesús Salamanca: "Pendulums on surfaces of constant curvature: an intrinsic approach"
• José Antonio Sánchez Pelegrín: "Uniqueness results for complete maximal hypersurfaces in certain Robertson-Walker spacetimes"
• Daniel Wysocky: "Structure and classification of Lie bialgebras"
• Marcin Zając: "The reduced mechanics on a Lie group"

	THURSDAY	FRIDAY	SATURDAY
9:00-9:30	Registration	A. Cabrera	M. Caponigro
9:30-10:30	A. Cabrera	A. Cabrera	M. Caponigro
10:30-11:00	A. Cabrera	Coffee + Poster	Coffee
11:00-11:30	Coffee	M. Palacios	W. Clark
11:30-13:00	N. Sansonetto	M. Caponigro	N. Sansonetto
15:00-15:30	V. Jiménez	A. Rojo	
15:30-16:30	M. Caponigro	N. Sansonetto	
16:30-17:00	Coffee	Coffee + Poster	
17:00-17:30	D. Bouloc	A. Cabrera	
17:30-18:00	M. Fontaine	A. Cabrera	
18:00-18:30	A. Shaddad	F. Boriero	

COURSES

INTRODUCTION TO SUPERGEOMETRY

Alejandro Cabrera

In this mini-course, we shall introduce basic concepts of supergeometry with an eye on (ordinary, non-super) practical geometric applications. The key point to be emphasized is that of supergeometry as a simplifying and unifying language for seemingly unrelated/complicated geometric structures and formulas. With this in mind, we shall first see some fundamentals concerning 'super' analogues of linear algebra and calculus, and then jump to supermanifolds together with supergeometric structures on them. These structures can enconde in a simple way Lie algebroids, Courant algebroids, etc, while supercalculus is a powerfull tool which can be used to study complicated formulas/theorems (like Mathai-Quillen representatives for characteristic classes of vector bundles).

STABILIZATION AND OPTIMAL CONTROL IN MULTIAGENT DYNAMICS

Marco Caponigro

In this lectures we consider classical results of feedback stabilization and optimal control and their applications in optimal stabilization of dissipative systems. Multiagent systems with their emergent behaviors represent a natural example of dissipative systems and they will be the motivating example for our lectures. We will see how classical results in control apply to this rather new research field opening new problems and offering new perspectives.

INTEGRABILITY AND NONHOLONOMIC SYSTEMS WITH SYMMETRY

Nicola Sansonetto

Nonholonomic mechanical systems are mechanical systems with (nonintegrable) constraints in the velocities. A first consequence of the presence of the constraints is the non variational origin of these systems. This fact has strong implications: for example, Noethers Theorem does not hold and they are not Hamiltonian, therefore all the well developed theory of integrable Hamiltonian systems does not apply. These aspects, among others, make nonholonomic systems extremely interesting, rich and challenging.
In the course we will first review some basic facts about integrability of Hamiltonian and non Hamiltonian systems, underlying the typical geometric features of the Hamiltonian framework that still survive in the non-Hamiltonian case.
Then, after an introduction to nonholonomic systems, we will analyze the failure of Noethers Theorem and its consequences from the point of view of integrability. Nevertheless we will discuss a method to obtain first integrals out of the symmetry of a given system, thus shedding some light to a possible relation between the existence of first integrals and the presence of symmetries in noholonomic mechanical systems.
Eventually we will apply this theory to integrable nonholonomic systems, illustrating the obtained results to concrete examples.

SHORT TALKS

Trajectory generation for under-actuated mechanical systems: the "limbo dance" problem

Fabrizio Boriero

November 11, 2016

In this communication we will propose an optimal solution of kinodynamic motion planning for a class of under-actuated systems. Inspired by [3] and [1], the aim of our work is to implement a method that allows to plan and generate an optimal trajectory for underactuated system (of super-articulated type [4]) avoiding a fixed (in space) obstacle and, at the same time, satisfying second-order constraints (also know as "kynodinamic constraints"). We apply the method to solve the "Limbo dance" problem for the cart-pole. More precisely, starting from a pre-assigned initial configuration, the cart pole has to reach a given final configuration with the pendulum that avoids an obstacle fixed at a certain (positive) high. The results presented are based on an on-going collaboration with Sansonetto & all (see e.g.[2])

References

[1] A.M. Bloch, Nonholonomic mechanics and control. Interdisciplinary Applied Mathematics, 24. Springer-Verlag, New York, 2003.

[2] F. Boriero, N. Sansonetto, A. Marigonda, R. Muratore and P. Fiorini, Optimal solutions of kinodynamic motion planning for the cart-pole system. Work in progress

[3] L. Colombo, D. Martin de Diego and M. Zucalli, Optimal control of underactuated mechanical systems: a geometric approach. J. Math. Phys. 51 (2010), no. 8, 083519, 24 pp.

[4] J. Baillieul, The Geometry of Controlled Mechanical Systems. Mathematical Control Theory. Springer, New York, 1999 , pp. 322–354.

Singular fibers of the Gelfand-Cetlin system on $\mathfrak{su}(n)$

Damien Bouloc

The classical Gelfand-Cetlin system is a completely integrable system on the coadjoint orbits of the Lie algebra $\mathfrak{su}(n)$ introduced by Guillemin and Sternberg. In this talk we will recall the definition of this system and show that its singular fibers are embedded submanifolds, which is quite a surprise regarding the fact that the system has degenerate singularities. The proof relies on a geometric interpretation of the fibers as sets of increasing sequences of ellipsoids in $\mathbb{C}^n$ with fixed radii. This is part of a joint work with Eva Miranda and Nguyen Tien Zung.

Speaker: William Clark (University of Michigan, Ann Arbor).

Title: Time minimal control for a quantum Hamiltonian system under Lindblad dissipation.

Abstract: We investigate the optimal Hamiltonian control of n=2 density operators under Lindblad dissipation as one moves away from the completely mixed state. Inside the Bloch ball there exists an ellipsoid (the escape chimney) where the purity derivative is positive on the inside. We compute time minimal control schemes that steer the completely mixed state to the apogee of the escape chimney (the point of maximal purity in the reachable set).

References:

B. Bonnard and D. Sugny. Geometric optimal control and two-level dissipation quantum systems. *Control and Cybernetics,* 38(4A):1053-1080, 2009.

P. Rooney, A. M. Bloch, and C. Rangan Phys. Rev. A 93, 063424 – Published 27 June 2016

W. Clark, A. Bloch, L. Colombo, and P. Rooney. Time optimal controls for 2-level quantum systems. Working paper, 2016.

Explicit symmetry breaking for equivariant Hamiltonian systems

Marine Fontaine

November 15, 2016

Short Abstract

Explicit symmetry breaking occurs when a dynamical system having a certain group of symmetries is perturbed in a way that the perturbation conserves only some symmetries of the original system. We give a geometric approach to study this phenomenon in the setting of equivariant Hamiltonian systems. A lower bound for the number of orbits of equilibria and orbits of relative equilibria that will persist after a small perturbation is given, provided we verify some non-degeneracy conditions. This work is in collaboration with James Montaldi.

Study of Cosserat media with Lie groupoids and algebroids

Víctor Jiménez

A Lie groupoid, called material Lie groupoid, is associated in a natural way to any Cosserat medium. The corresponding Lie algebroid, called material Lie algebroid, is used to characterize the homogeneity property of the material. We also relate these results with the previously ones in terms of non-holonomic second-order $\overline{G}$-structures.

A variational formalism for finite dimensional thermo-mechanical systems.

Maximiliano A. Palacios Amaya

CONICET - Centro Atómico Bariloche, UNComahue.

Abstract:
The aim of this short talk is to introduce a class of physical systems that combine a finite number of mechanical and thermodynamic observables, the *finite dimensional thermo-mechanical systems*, by means of some motivational examples. Briefly berofehand, the main mathematical features of thermodynamic (quasi-static) finite dimensional systems will be introduced. The evolution equations of the involved observables are obtained in each example by using, essentially, the Newton's law and the First Law of Thermodynamics only. It will be shown that such equations are similar to those defining certain mechanical systems with higher order constraints. Moreover, it will be shown how such systems can be described in a variational formalism using second order constrains.

Discrete moving Frames and Noether's Finite Difference Conservation Laws

Ana Rojo

Discrete moving frames have proved to have a large spectrum of applications as well as leading to improvement regarding computations. Further, the theory of discrete moving frames provides an useful tool when solving variational problems or in the study of integrable systems. In this talk we will consider discrete finite difference Lagrangians which are invariant under a variety of group actions. We write the Euler-Lagrange difference equations and the Noether's difference conservation laws in terms of the invariants of the action and a discrete moving frame. The appearance of the moving frame in the expression for the conservation laws makes explicit the equivariance of the laws under the group action. We exhibit how the use of the discrete moving frame eases the integration problem.

Point Vortices on the Complex Projective plane

Amna Shaddad

A vortex (a whirlwind of fluid) modelled as a coordinate point with an associated 'weight' which corresponds to the strength of that vortex is a point vortex. There has been a lot of interest in modelling systems of point vortices on 2-spheres: we take this a step further and introduce results from modelling point vortices on $\mathbb{CP}^2$ with weighted symplectic form. This system has a natural $SU(3)$ symmetry, and the geometry of this non-abelian momentum map is shown via the momentum polytope, which is a tool for exploring, amongst other things, the dynamics (or relative equilibria) of the mechanical system. Based on the weighting ratios between each of the point vortices in the system of vortices, the resulting momentum polytopes are classified into different types. This talk will introduce these results and the dynamics of the different point vortex systems.