

# The N-body problem in spaces of constant curvature

Florin Diacu  
University of Victoria, Canada

The classical gravitational N-body problem was first introduced by Newton in 1687. The idea of extending gravitation from the Euclidean space (which has zero Gaussian curvature) to spaces of non-zero constant Gaussian curvature occurred independently to Bolyai and Lobachevsky in the 1830s, but started developing only decades later. Most results in this direction concerned only the case  $N=2$ . Recently, however, we derived the equations of motion in the general case and started a systematic study of the singularities that occur in the equations of motion, of relative equilibria, i.e. orbits whose mutual distances remain constant in time, and of rotopulsators, i.e. orbits that perform rotations and dilations or contractions at the same time, [1,2,3,4,5,6,7,8]. The goal of these lectures is to introduce the problem, derive the equations of motion, and present some of the most interesting results obtained so far on the N-body problem in spaces of non-zero constant curvature.

## References:

- [1] F. Diacu, On the singularities of the curved N-body problem, *Trans. Amer. Math. Soc.* 363, 4 (2011), 2249--2264.
- [2] F. Diacu, Polygonal homographic orbits of the curved 3-body problem, *Trans. Amer. Math. Soc.* 364 (2012), 2783--2802.
- [3] F. Diacu, Relative equilibria of the curved N-body problem, *Atlantis Studies in Dynamical Systems*, vol. 1, Atlantis Press, Amsterdam, 2012.
- [4] F. Diacu, Relative equilibria of the 3-dimensional curved N-body problem, *Memoirs Amer. Math. Soc.* 228, 1071 (2013).
- [5] F. Diacu and S. Kordlou, Rotopulsators of the curved N-body problem, *J. Differential Equations* 255 (2013) 2709--2750.

[6] F. Diacu, The curved N-body problem: risks and rewards, *Math. Intelligencer* 35, 3 (2013), 24--33.

[7] F. Diacu, E. Pérez-Chavela, and M. Santoprete, The N-body problem in spaces of constant curvature. Part I: Relative equilibria, *J. Nonlinear Sci.* 22, 2 (2012), 247–266.

[8] F. Diacu, E. Pérez-Chavela, and M. Santoprete, The N-body problem in spaces of constant curvature. Part II: Singularities, *J. Nonlinear Sci.* 22, 2 (2012), 267–275.