

## Where to find Morse families in geometric mechanics?

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It is well-known in the area of symplectic geometry that “everything is a Lagrangian submanifold” [1]. These submanifolds have been extensively used in geometric mechanics to describe Lagrangian and Hamiltonian dynamics [2, 3], Hamilton-Jacobi equations, Helmholtz conditions, etc. Recently, interest in Morse families [4], which define particular Lagrangian submanifolds, has appeared because they also contribute to the understanding of geometric mechanics. For instance, in control theory Morse families provide a geometric characterization of necessary conditions for optimality [5] under some particular assumptions. Moreover, Dirac structures [6] accompanied by a Morse family describes a geometric framework to characterize interconnection of mechanical and control systems [7].

In this talk, we show how Morse families naturally appear in geometric mechanics and geometric control theory.

\*(joint work with H. Cendra (U. Nacional del Sur, Argentina), D. Iglesias Ponte (ULL, Spain), D. Martín de Diego (ICMAT, Spain)).

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