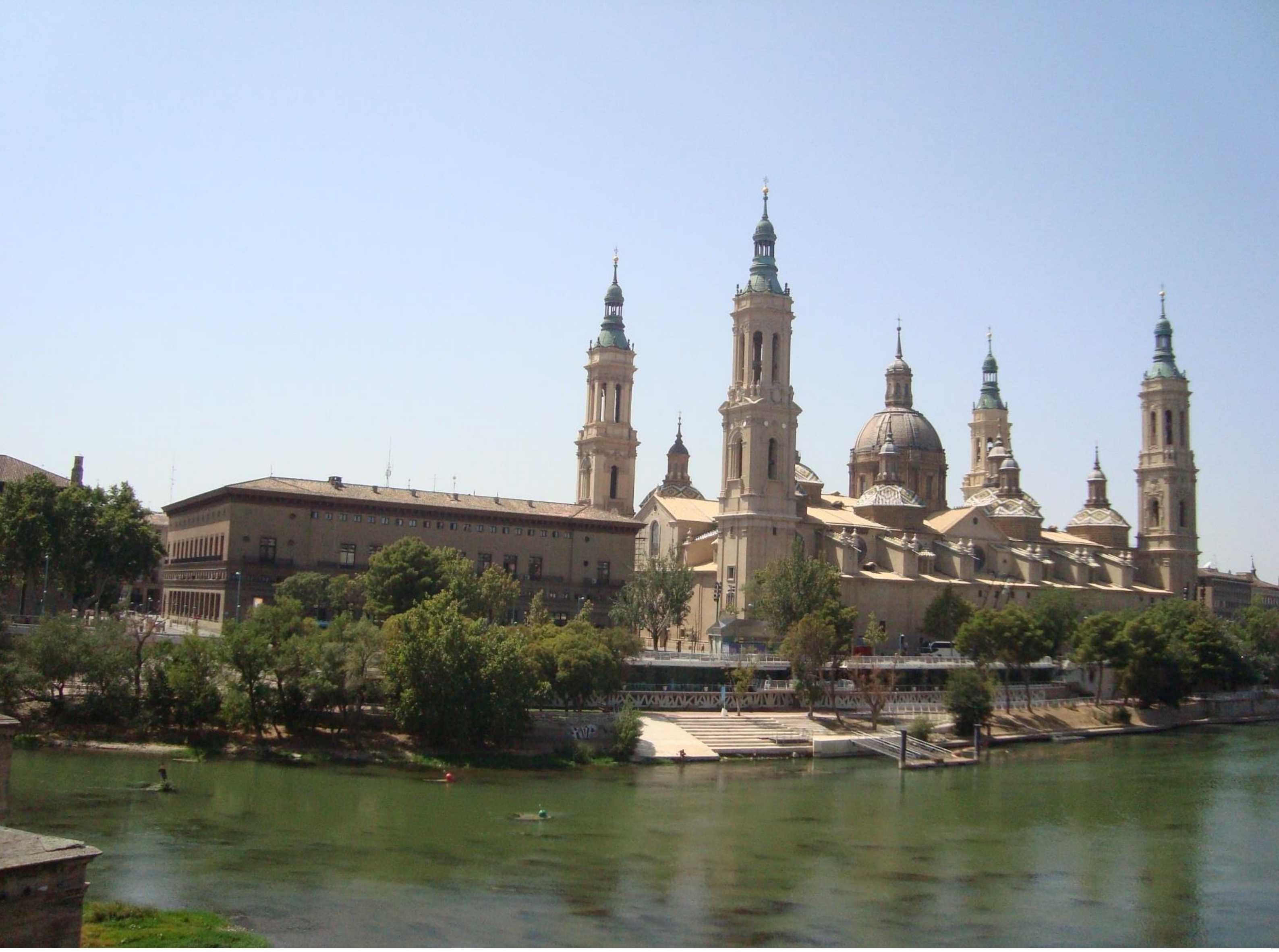

9th INTERNATIONAL YOUNG RESEARCHERS WORKSHOP ON GEOMETRY, MECHANICS AND CONTROL

January 19 – 21, 2015

Zaragoza, Spain

http://gmcnet.webs.ull.es/?q=activity-detaill/1284

Universidad Zaragoza

9:00 - 9:30	Registration		
9:30 - 11:00	T. Mestdag	R. Ghezzi	D. Martínez
11:00 - 11:30	Coffee	Coffee	Coffee
11:30 - 13:00	R. Ghezzi	D. Martínez	T. Mestdag
13:00 - 13:30	M. Rotkiewicz	M. Farré	G. Waeyaert
13:30 - 15:30	Lunch time	Lunch time	Lunch time
15:30 - 16:30	D. Martínez	T. Mestdag	R. Ghezzi
16:30 - 17:00	Coffee	Coffee	Coffee
17:00 - 17:30	H. Jolany	Poster Session	N. Sansonetto
17:30 - 18:00	J. Jover		A. Arnaudon
18:00 - 18:30	T. Milkovszki	I. Gheorghiu	N. Martínez

1. **Minicourses**

Geometry and Analysis on Almost-Riemannian Manifolds.

Roberta Ghezzi
University of Burgundy, Dijon, France

An almost-Riemannian structure on a n-manifold is a generalized Riemannian structure whose local orthonormal frames are given by Lie bracket generating n-tuple of vector fields that can become collinear. The distribution generated locally by orthonormal frames has maximal rank at almost every point, but in general it has rank $<$ n on a nonempty set which is generically an embedded submanifold. The first example is the Grushin space, which appeared in the context of hypoelliptic operators in the 70s. More recently, almost-Riemannian geometry arise as the natural framework to model problems of population transfer in quantum control systems. We will investigate topological, metric and geometric aspects of almost-Riemannian manifolds and we will present recent results concerning diffusion processes. Time permitting, we will give the spectral analysis of the Laplace-Beltrami operator.

References

[1] A.A. Agrachev, U. Boscain, G. Charlot, R. Ghezzi, M. Sigalotti, Two-Dimensional Almost-Riemannian Structures with Tangency Points, Annales de lInstitut Henri Poincaré (C) Analyse Non Linéaire, 27(3) (2010) pp. 793-807.
[2] U. Boscain, G. Charlot, M. Gaye, P. Mason, Local properties of almost-Riemannian structures in dimension 3, arXiv:1407.0610
[3] U. Boscain, C. Laurent, The Laplace-Beltrami operator in almost-Riemannian geometry, Annales Institut Fourier (Grenoble) 63 (2013), no. 5, 1739-1770.
[4] U. Boscain, D. Prandi, M. Seri, Spectral Analysis and the Aharonov-Bohm effect on certain almost-Riemannian manifolds, arXiv: 1406.6578

Geometry and topology of coadjoint orbits

David Martínez Torres
PUC-Rio, Brazil

Starting from Hamiltonian actions, we shall look at all coadjoint orbits (Poisson geometry) to obtain a geometric incarnation of basic facts from Lie theory of compact Lie groups. We shall also look at coadjoint orbits of compact groups individually (symplectic geometry), and discuss their topology and complex geometry. Time permitting, we shall recall connections with representation theory, and discuss features of coadjoint orbits of non-compact semisimple Lie groups.

The inverse problem of the calculus of variations

Tom Mestdag
Ghent University, Belgium

The inverse problem we will discuss is the following one: given a system of second-order ordinary differential equations, under what circumstances can these equations be derived from a variational principle, i.e. when does there exist a regular Lagrangian function, such that its corresponding Euler-Lagrange equations are equivalent to the original equations (i.e. have the same solutions). In the introduction to his famous paper 'Solution of the inverse problem of the calculus of variations', published in 1941, Fields medalist Jesse Douglas said that the 'problem indicated in the title is one of the most important hitherto unsolved problems of the calculus of variations'. Unlike his title suggests, Douglas gave the solution to this problem for two dimensions only, and the problem has not been solved (in the sense that Douglas solved it) for higher dimensions, either in general, or even in any particular subcase. This is not to say that no progress has been made in the intervening 70 years.

In the lectures, we will make use of a differential geometric approach to the problem, the calculus of derivations of forms along a map. We will focus on some integrability aspects, on some specific subcases, on an extension of the problem to systems with non-conservative forces, and on the related problem of metrizability in Finsler geometry.

Some general references to the inverse problem are:

References

[1] I. M. Anderson and G. Thompson, The inverse problem of the calculus of variations for ordinary differential equations, Mem. Amer. Math. Soc. 98 (1992).

[2] J. Grifone and Z. Muzsnay, Variational Principles for Second-order Differential Equations, (World Scientific 2000).

[3] O. Krupkova and G. E. Prince, Second order ordinary differential equations in jet bundles and the inverse problem of the calculus of variations, in: Handbook of Global Analysis, D. Krupka and D. Saunders eds. (Elsevier 2008) 837904.

[4] R. M. Santilli, Foundations of Theoretical Mechanics I. The Inverse Problem in Newtonian Mechanics" (Spinger-Verlag 1978)

[5] W. Sarlet, G. Thompson and G.E. Prince, The inverse problem in the calculus of variations: the use of geometrical calculus in Douglas's analysis, Trans. Amer. Math. Soc. 354, 2897-2919 (2002)

2. **Short talks**

Lagrangian reduction for completely integrable systems

Alexis Arnaudon
Imperial College London

In this talk I will present a novel approach for completely integrable systems such as NLS, KdV, CH etc... I will use and extend the multi-time concept, first introduced almost 40 years ago by the Japanese school and later developed by Flaschka, Newell and Ratiu [1], to place integrable hierarchies of PDEs into the setting of Lagrangian reduction by symmetry. The difference with the standard Hamiltonian theory is rather small but turns out to be very important. This new formulation will lead to a notion of weak integrability through the use of a Sobolev norm. For example, the Camassa-Holm equation is among this class of integrable systems. New weakly completely integrable equations will also be presented.

References

[1] H. Flaschka, A. C. Newell and T. Ratiu, Physica **9D**, 300 (1983).

Inverse problem in discrete mechanics

Marta Farré Puiggalí
Instituto de Ciencias Matemáticas

I plan to give a geometric description of the inverse problem of the calculus of variations in the case of discrete mechanics following the ideas for the continuous setting given in [1]. The results can be useful to compare the qualitative behaviour of numerical methods for second order differential equations.

References

[1] M. Barbero-Liñán, M. Farré Puiggalí and D. Martín de Diego. Isotropic submanifolds and the inverse problem for mechanical constrained systems : arXiv:1404.1961

The virial theorem for nonholonomic systems

Irina Mihaela Gheorghiu
Universidad de Zaragoza

Leaving from the geometric approach for the generalized virial theorem given in [Cariñena J.F., Falceto F. and Rañada M.F., A geometric approach to a generalized virial theorem, *J. Phys. A: Math. Theor,* **45**, 395210 (19pp) (2012)], we present it for Lagrangian systems of mechanical type on a Riemannian manifold , and for special cases of virial functions, making use of the properties of the Killing vector field associated to the Riemannian structure. We also generalize the virial theorem to mechanical systems on Lie algebroids, and to nonholonomic systems both on the tangent bundle and on Lie algebroids using two approaches. We will make use of the quasi-coordinates to write these instances of the virial theorem.

Song-Tian theory in Birational Geometry and MMP

Hassan Jolany
University of Lille 1

Recently Gang Tian introduced a new method in Minimal model program by using Kahler Ricci flow theory. He proposed a program of finding canonical metrics on canonical models of projective varieties of positive Kodaira dimension. They also gave an affirmative answer for finding canonical metric for complex surfaces by using modified Kahler Ricci flow. He introduced generalized Kahler Einstein Metric by using Weil-Petersson metric on moduli space of Calabi-Yau manifolds and considered the existence of generalized Kahler Einstein metrics. I try to explain this new theory.

Kähler-Lie systems and geometric quantum mechanics

Jorge Alberto Jover Galtier
University of Zaragoza

In this seminar, we will study the definition and properties of Kähler-Lie systems and its application to geometric quantum mechanics. Firstly, we will introduce the geometric description of quantum mechanics and its usage to the study of dynamics on finite-dimensional complex manifolds. We will observe that this dynamics is described by a type of Lie systems whose Vessiot-Guldberg Lie algebra is composed of holomorphic vector fields with respect to a Kähler structure. This justifies the definition of a new class of Lie systems on Kähler manifolds with rich geometric features: the Kähler-Lie systems. Secondly, we aim to present the geometric methodology developed to study their superposition rules, time independent constants of motion, Lie symmetries, etc. Finally, we will apply this methodology to several practical problems in geometric quantum mechanics.

Higher-Poisson structures, reduction and Field Theory

Nicolas Martinez
IMPA

From multi-symplectic and poly-symplectic forms is possible to define the notion of higher-symplectic groupoids and induces the higher-Poisson structures as Poisson structures are obtained by symplectic groupoids. Symmetries by Lie groups action on such structures will be considered and give its reduction. At the end I will show how these structures appears naturally in the Field Theory framework.

Invariant metrizability and projective metrizability on Lie groups

Tamás Milkovszki
University of Debrecen

In this talk we examine the invariant metrizability and projective metrizability problem for the special case of the geodesic flow associated to the canonical symmetric invariant connection of a Lie group. We prove that the canonical connection is projectively Finsler metrizable if and only if it is Riemann metrizable. That means that the structure is rigid in the sense that considering left-invariant metrics, the potentially much larger class of projective Finsler metrizable sprays, corresponding to Lie groups, coincides with the class of Riemmann metrizable sprays.

Prototypes of higher algebroids with applications to variational calculus

Mikolaj Rotkiewicz
Warsaw University

Reductions of higher tangent bundles of Lie groupoids provide natural examples of geometric structures which we would like to call higher algebroids. The basic problem is: what is the algebraic structure on the reduced bundle inherited from groupoid multiplication? The key difficulty is that there is no bracket operation on the space of sections of a higher tangent bundle. The basic idea which allowed us to solve the problem is a reformulation of a definition of an algebroid in terms of a relation κ which can be obtained by a reduction of the canonical involution of TTG if the algebroid integrates to a Lie groupoid G. Thus a higher algebroid is, in principle, a graded bundle equipped with a relation of special kind. Such a point of view is natural in Geometric Mechanics higher-order systems with internal symmetries can be reduced to systems defined on such higher algebroids.

Complete integrability, the Hamilton-Jacobi equation and nonholonomic systems

Nicola Sansonetto
Universita di Padova

The Hamilton-Jacobi theory is at the core of the symplectic and variational structure of Hamiltonian mechanics. There have been a large number of extensions of this theory to systems with nonholonomic constraints. In Hamiltonian mechanics there are two versions of the (time independent) HamiltonJacobi equation. A weaker version is related to the existence of an integral of the equation, and corresponds to the existence of an invariant Lagrangian submanifold. A stronger version is related to the existence of a complete integral of the equation, and corresponds to the complete integrability, in the sense of Liouville-Arnold, of the system. All the mentioned extensions to systems with nonholonomic constraints refer to the weaker of the two versions of the HamiltonJacobi theory, but several of these references point out, or at least mention, the interest of understanding the possible relations, in the nonholonomic context, between integrability and Hamilton-Jacobi theory. To our knowledge, however, there are no discussions of this relationship, nor clarifications of its (im)possibility and/or limitations. The aim of this work is to provide such an analysis. In terms that need to be precised, our conclusion is that this relation fails. At the most primitive level, this is due to the failure, outside the Hamiltonian context, of the equality of conservation laws and symmetries, which is encoded in the interplay between dynamics and geometry so peculiar to the Hamiltonian case. As a rule, outside the variational world, symmetries do not provide integrals of motion, and outside the symplectic world, integrals of motion do not generate symmetries. The remnants of symplecticity that persist to the nonholonomic world are not enough to save this situation and this makes the link symmetry-integrability, if any, different from that of the Hamiltonian case.

Lifted tensors and Hamilton-Jacobi separability

Goedele Waeyaert
Ghent University

We will discuss natural lifting operations from a bundle over R to the dual of the first jet bundle, which is the appropriate manifold for the geometric description of time-dependent Hamiltonian systems. The main purpose is to define a complete lift of a type (1,1) tensor field on the base manifold. This construction and its properties are, in particular, relevant for an intrinsic characterization of Forbat's conditions for separability of the time-dependent Hamilton-Jacobi equation.

References

[1] G. Waeyaert and W. Sarlet, Lifted tensors and Hamilton-Jacobi separability, J. Geom. Phys. 86 (2014), 122-133.

3. **Posters**

Strings in general relativity

Octavian Postavaru
ELI-NP Bucharest and University of Bucharest

Quantum gravity is suppose to arise from a unification between relativity and quantum theory. There are many paths to do this: strings, loops, twistors, non-commutative geometry and topi. Strings are an ensemble of particles with springs between them, and they are very interesting objects which play a very important role in the quantum gravity. Open strings are known as photons, and closed strings as gravitons, both being quantum mechanical objects. Black Hole (BH) is the most doll object in the Universe, but it is not dead (infinitely cold). The name Black could came from the fact that the object does absorb all light waves, or does emit black-body (thermal) radiation. To say that a system has entropy is another way to say that there are a large number of microscopic degrees of freedom which are too small and too numerous for us to keep truck of them. In particular, this problem arouse in general relativity, for BH description. When Beckenstein put forward the idea that the BHs have entropy, the natural question was what are this tinny microscopic things which apparently are at the surface of the BH (entropy is proportional to the BH area). String theory can provide an explanation of the entropy of Bhs.

References

[1] The Physics of the Black Holes, O. Postavaru, Lambert 2014

Geometric Inequalities for Sasakian Space Forms

Ileana Presura
University of Bucharest

Legendrian and special contact slant submanifolds in Sasakian space forms play an important role in contact geometry. This article has two parts. In the first part of it, we obtain Chen like inequalities for legendrian submanifolds in Sasakian space forms, i.e relationships between intrinsic and intrinsic invariants of such submanifolds, involving the scalar curvature and Chen first invariant, respectively, and the squared mean curvature and the ϕ-sectional curvature of the ambient space. In the last part, we obtain Chen like inequalities for a special contact slant submanifold M in a Sasakian space form $\widetilde{M}(c)$, in terms of the main extrinsic invariant, namely the squared mean curvature.

Applications of Lie–Hamilton systems on the plane: Cayley–Klein Riccati equations and beyond

Cristina Sardón
Universidad de Salamanca

A Lie–Hamilton system is a nonautonomous system of first-order ordinary differential equations describing the integral curves of a t-dependent vector field taking values in a finite-dimensional real Lie algebra of Hamiltonian vector fields with respect to a Poisson structure. After reviewing the classification of finite-dimensional real Lie algebras of Hamiltonian vector fields on $\mathbb{R}^2$, we present new Lie–Hamilton systems on the plane with physical, biological and mathematical applications. New results cover Cayley–Klein Riccati equations, the hereafter called planar diffusion Riccati systems and complex Bernoulli equations, all of them with t-dependent real coefficients. Furthermore, we study the existence of local diffeomorphisms among new and already known Lie–Hamilton systems on the plane. In particular, we show that the Cayley–Klein Riccati equations describe as particular cases well-known coupled Riccati equations, second-order Kummer–Schwarz equations, Milne–Pinney equations, the harmonic oscillator with t-dependent frequency and other systems of physical and mathematical relevance.

The dynamics' n-body problem: An example for 2-body

Rodrigo Schaefer
Universitat Politécnica de Catalunya

This work is based on the paper "The n-body, the n-vortices and the n-charges' dynamics on surfaces : a common view point" (Boatto, Dritschel and Schaefer) (in preparation), studying particular cases and consequences of them. The work can be divided in the following parts:

- N-body dynamics' formulations utilizing the surface's intrinsic geometry on the movement is restricted.
- Comparison, 2-body problem on plane case, between usual and proposal formulation, evidencing the difference of the results about the orbits.
- Study of the system's reduction dimensional process on the cylinder and sphere cases for 2-body problem. Introduction to the mass metric notion.
- Demonstration of the Bertrand's theorem (about conditions for closed orbits) for revolution surfaces and consequences, and a generalization for Laplace-Runge-Lenz vector based respectively on the articles "Gravitational and harmonic oscillator potentials on surfaces of revolution" and "Block regularization of the kepler problem on surfaces of revolution with positive constant curvature". Both of Santoprete, M .

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This work was my dissertation in master (in Brazil with orientation of Stefanella Boatto).