Motion Planning via Reconstruction Theory

M. Zoppello[*], F. Fassó[*], S. Passarella[*]

Abstract

In Geometric Control Theory, problems of motion planning consist in finding a control that steers a control system from a starting configuration to a prescribed final one. This problems are common in several fields of research: biology, chemistry, medicine, robotics. In this work we present an approach to study these problems based on the techniques of reconstruction of dynamical systems with symmetry. In particular we focus on a class of control problems, that we called robotic locomotion systems, which are driftless affine control systems with configuration space the total space of a trivial principal ber bundle $\pi : G \times S \to S$, with G a Lie group and S the shape space which is a manifold [4]. The problem is, for every assigned loop in the base space, to determine the motion of the system on the fiber over the considered loop. This problem is linked to the one of reconstruction in dynamical systems. The main difference is that now the curve on the base manifold is not the integral curve of a (reduced) differential equation, but it is assigned by the controller. The theory of reconstruction under the action of a connected Lie group is well studied from different points of view [5]. Our purpose is to the importance in motion planning of reconstruction results. In particular there are two cases: if the group is compact, the motion on the fiber over a loop is quasi-periodic [3]; if the group is not compact, the motion on the ber over a loop is either quasi-periodic or there is a drift [1]. The generic case is determined by the group itself. As a consequence some relevant information on the possible motion can be a priori inferred from the structure of the group alone. We present some examples involving some classical Lie groups: $SE(2)$, $SO(2) \times \mathbb{R}^2$, $SE(3)$, $SO(3) \times \mathbb{R}^3$.

References

[1] Ashwin P. , Melbourne I.
Noncompact drift for relative equilibria and relative periodic orbits
IOP Publishing Ltd and LMS Publishing Ltd, 1997.

[2] Fassó F., Giacobbe A.
Geometry of invariant tori of certain integrable systems with symmetry and application to a nonholonomic system
Sigma 2007.

[3] Field M.J.
Equivariant dynamical systems
Trans. Am. Math. Soc. 259, 1980, p. 185- 205.

[4] Kelly S.D. and Murray R.M.
Geometric phases and robotic locomotion
Journal of Robotic Systems, 12(6), 417-431, 1995.

[5] Marsden J.E., Montgomery R., Ratiu
Reduction, symmetry and phases in mechanics
American Mathematical Society, 1990.

[*]Universitá degli studi di Padova